JEE–MAIN EXAMINATION – JANUARY 2025

(HELD ON WEDNESDAY 22nd JANUARY 2025)

TIME : 9:00 AM TO 12:00 NOON

MATHEMATICS

SECTION-A

1. The number of non-empty equivalence relations on the set {1,2,3} is :

(1) 6 (2) 7

(3) 5 (4) 4

Ans. (3)

Sol. Let R be the required relation

$A = \{(1, 1)\ (2, 2), (3, 3)\}$

(i) $|R| = 3$, when $R = A$

(ii) $|R| = 5$, e.g. $R = A \cup \{(1, 2), (2, 1)\}$

Number of R can be [3]

(iii) $R = \{1, 2, 3\} \times \{1, 2, 3\}$

Ans. (5)

2. Let $f : \mathbf{R} \to \mathbf{R}$ be a twice differentiable function such that $f(x + y) = f(x)\ f(y)$ for all $x, y \in \mathbf{R}$. If $f'(0) = 4a$ and f staisfies $f''(x) - 3a\, f'(x) - f(x) = 0$, $a > 0$, then the area of the region

$R = \{(x,y) \mid 0 \le y \le f(ax), 0 \le x \le 2\}$ is :

(1) $e^2 - 1$ (2) $e^4 + 1$

(3) $e^4 - 1$ (4) $e^2 + 1$

Ans. (1)

Sol. $f(x + y) = f(x).f(y)$

$\Rightarrow f(x) = e^{\lambda x}\quad f'(0) = 4a$

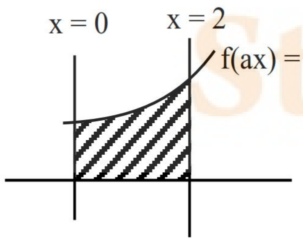
$\Rightarrow f'(x) = \lambda e^{\lambda x} \Rightarrow \lambda = 4a$

So, $f(x) = e^{4ax}$

$f''(x) - 3af'(x) - f(x) = 0$

$\Rightarrow \lambda^2 - 3a\lambda - 1 = 0$

$\Rightarrow 16a^2 - 12a^2 - 1 = 0 \Rightarrow 4a^2 = 1 \Rightarrow \boxed{a = \frac{1}{2}}$



$F(x) = e^{2x}$

$\text{Area} = \int_0^2 e^x dx = \boxed{e^2 - 1}$

3. Let the triangle PQR be the image of the triangle with vertices (1,3), (3,1) and (2, 4) in the line $x + 2y = 2$. If the centroid of ΔPQR is the point (α, β), then $15(\alpha - \beta)$ is equal to :

(1) 24 (2) 19

(3) 21 (4) 22

Ans. (4)

Sol. Let 'G' be the centroid of Δ formed by (1, 3) (3, 1) & (2, 4)

$G \cong \left(2, \frac{8}{3}\right)$

Image of G w.r.t. $x + 2y - 2 = 0$

$$\frac{\alpha - 2}{1} = \frac{\beta - \frac{8}{3}}{2} = -2\frac{\left(2 + \frac{16}{3} - 2\right)}{1+4}$$

$$= \frac{-2}{5}\left(\frac{16}{3}\right)$$

$$\Rightarrow \alpha = \frac{-32}{15} + 2 = \frac{-2}{15},\ \beta = \frac{-32 \times 2}{15} + \frac{8}{3} = \frac{-24}{15}$$

$15(\alpha - \beta) = -2 + 24 = 22$

4. Let z_1, z_2 and z_3 be three complex numbers on the circle $|z| = 1$ with $\arg(z_1) = \frac{-\pi}{4}$, $\arg(z_2) = 0$ and $\arg(z_3) = \frac{\pi}{4}$.

If $|z_1\bar{z}_2 + z_2\bar{z}_3 + z_3\bar{z}_1|^2 = \alpha + \beta\sqrt{2}$, $\alpha, \beta \in Z$, then the value of $\alpha^2 + \beta^2$ is :

(1) 24 (2) 41

(3) 31 (4) 29

Ans. (4)

Sol. $Z_1 = e^{-i\pi/4}$, $Z_2 = 1$, $Z_3 = e^{i\pi/4}$

$$|z_1\bar{z}_2 + z_2\bar{z}_3 + z_3\bar{z}_1|^2 = \left|e^{-i\frac{\pi}{4}} \times 1 + 1 \times e^{-i\frac{\pi}{4}} + e^{i\frac{\pi}{4}} \times e^{i\frac{\pi}{4}}\right|^2$$

$$\left|e^{-i\frac{\pi}{4}} + e^{-i\frac{\pi}{4}} + e^{i\frac{\pi}{4}}\right|^2$$

$$= \left|2e^{-i\frac{\pi}{4}} + i\right|^2 = \left|\sqrt{2} - \sqrt{2}i + i\right|^2$$

$$= \left(\sqrt{2}\right)^2 + \left(1 - \sqrt{2}\right)^2 = 2 + 1 + 2 - 2\sqrt{2} = 5 - 2\sqrt{2}$$

$\alpha = 5, \beta = -2$

$\Rightarrow \alpha^2 + \beta^2 = 29$

5. Using the principal values of the inverse trigonometric functions the sum of the maximum and the minimum values of $16((\sec^{-1}x)^2 + (\text{cosec}^{-1}x)^2)$ is :

(1) $24\pi^2$ (2) $18\pi^2$

(3) $31\pi^2$ (4) $22\pi^2$

Ans. **(4)**

Sol. $16(\sec^{-1}x)^2 + (\text{cosec}^{-1}x)^2$

$\text{Sec}^{-1}x = a \in [0, \pi] - \left\{\frac{\pi}{2}\right\}$

$\text{cosec}^{-1}x = \frac{\pi}{2} - a$

$= 16\left[a^2 + \left(\frac{\pi}{2} - a\right)^2\right] = 16\left[2a^2 - \pi a + \frac{\pi^2}{4}\right]$

$\max]_{a=\pi} = 16[2\pi^2 - \pi^2 + \pi\frac{2}{4}] = 20\pi^2$

$\min]_{a=\frac{\pi}{4}} = 16\left[\frac{2\times\pi^2}{16} - \frac{\pi^2}{4} + \frac{\pi^2}{4}\right] = 2\pi^2$

Sum $= 22\pi^2$

6. A coin is tossed three times. Let X denote the number of times a tail follows a head. If μ and σ^2 denote the mean and variance of X, then the value of $64(\mu + \sigma^2)$ is :

(1) 51 (2) 48

(3) 32 (4) 64

Ans. **(2)**

Sol. $HHH \to 0$

$HHT \to 0$

$HTH \to 1$

$HTT \to 0$

$THH \to 1$

$THT \to 1$

$TTH \to 1$

$TTT \to 0$

Probability distribution

x_i	0	1
$P(x_i)$	$1/2$	$1/2$

$\mu = \sum x_i p_i = \frac{1}{2}$

$\sigma^2 = \sum x_i^2 p_i - \mu^2$

$= \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

$64(\mu + \sigma^2) = 64\left(\frac{1}{2} + \frac{1}{4}\right) = 48$

7. Let $a_1, a_2, a_3.....$ be a G.P. of increasing positive terms. If $a_1a_5 = 28$ and $a_2 + a_4 = 29$, the a_6 is equal to

(1) 628 (2) 526

(3) 784 (4) 812

Ans. **(3)**

Sol. $a_1.a_5 = 28 \Rightarrow a.ar^4 = 28 \Rightarrow a^2r^4 = 28$ ...(1)

$a_2 + a_4 = 29 \Rightarrow ar + ar^3 = 29$

$\Rightarrow ar(1 + r^2) = 29$

$\Rightarrow a^2r^2(1 + r^2)^2 = (29)^2$ ...(2)

By Eq. (1) & (2)

$\frac{r^2}{(1+r^2)^2} = \frac{28}{29\times 29}$

$\Rightarrow \frac{r}{1+r^2} = \frac{\sqrt{28}}{29} \Rightarrow r = \sqrt{28}$

$\because a^2r^4 = 28 \Rightarrow a^2 \times (28)^2 = 28$

$\Rightarrow a = \frac{1}{\sqrt{28}}$

$\therefore a_6 = ar^5 = \frac{1}{\sqrt{28}} \times (28)^2\sqrt{28} = 784$

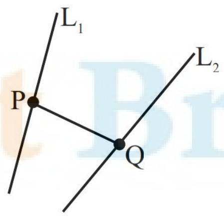
8. Let $L_1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $L_2 : \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ be two lines. Then which of the following points lies on the line of the shortest distance between L_1 and L_2 ?

(1) $\left(-\frac{5}{3}, -7, 1\right)$ (2) $\left(2, 3, \frac{1}{3}\right)$

(3) $\left(\frac{8}{3}, -1, \frac{1}{3}\right)$ (4) $\left(\frac{14}{3}, -3, \frac{22}{3}\right)$

Ans. **(4)**

Sol.



$P(2\lambda + 1, 3\lambda + 2, 4\lambda + 3)$ on L_1

$Q(3\mu + 2, 4\mu + 4, 5\mu + 5)$ on L_2

Dr's of $PQ = 3\mu - 2\lambda + 1, 4\mu - 3\lambda + 2, 5\mu - 4\lambda + 2$

$PQ \perp L_1$

$\Rightarrow (3\mu - 2\lambda + 1)2 + (4\mu - 3\lambda + 2)3 + (5\mu - 4\lambda + 2)4 = 0$

$38\mu - 29\lambda + 16 = 0 \quad ...(1)$

$PQ \perp L_2$

$\Rightarrow (3\mu - 2\lambda + 1)3 + (4\mu - 3\lambda + 2)4 + (5\mu - 4\lambda + 2)5 = 0$

$50\mu - 38\lambda + 21 = 0 \quad ...(2)$

By (1) & (2)

$\lambda = \frac{1}{3};\ \mu = \frac{-1}{6}$

$\therefore\ P\left(\frac{5}{3}, 3, \frac{13}{3}\right)$ & $Q\left(\frac{3}{2}, \frac{10}{3}, \frac{25}{6}\right)$

Line PQ

$$\frac{x - \frac{5}{3}}{\frac{1}{6}} \quad \frac{y-3}{\frac{-1}{3}} \quad \frac{z - \frac{13}{3}}{\frac{1}{6}}$$

$$\frac{x - \frac{5}{3}}{1} = \frac{y-3}{-2} = \frac{z - \frac{13}{3}}{1}$$

Point $\left(\frac{14}{3}, -3, \frac{22}{3}\right)$

lies on the line PQ

9. The product of all solutions of the equation $e^{5(\log_e x)^2 + 3} = x^8$, $x > 0$, is :

(1) $e^{8/5}$ (2) $e^{6/5}$

(3) e^2 (4) e

Ans. (1)

Sol. $e^{5(\ell nx)^2 + 3} = x^8$

$\Rightarrow \ell n\, e^{5(\ell nx)^2 + 3} = \ell n\, x^8$

$\Rightarrow 5(\ell nx)^2 + 3 = 8\ell nx$

$(\ell nx = t)$

$\Rightarrow 5t^2 - 8t + 3 = 0$

$t_1 + t_2 = \frac{8}{5}$

$\ell nx_1x_2 = \frac{8}{5}$

$x_1x_2 = e^{8/5}$

10. If $\sum_{r=1}^{n} T_r = \frac{(2n-1)(2n+1)(2n+3)(2n+5)}{64}$, then $\lim_{n\to\infty} \sum_{r=1}^{n} \left(\frac{1}{T_r}\right)$ is equal to :

(1) 1 (2) 0

(3) $\frac{2}{3}$ (4) $\frac{1}{3}$

Ans. (3)

Sol. $T_n = S_n - S_{n-1}$

$\Rightarrow T_n = \frac{1}{8}(2n - 1)(2n + 1)(2n + 3)$

$\Rightarrow \frac{1}{T_n} = \frac{8}{(2n-1)(2n+1)(2n+3)}$

$\lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{T_r} = \lim_{n\to\infty} 8 \sum_{r=1}^{n} \frac{1}{(2n-1)(2n+1)(2n+3)}$

$= \lim_{n\to\infty} \frac{8}{4} \sum \left(\frac{1}{(2n-1)(2n+1)} - \frac{1}{(2n+1)(2n+3)}\right)$

$= \lim_{n\to\infty} 2\left[\left(\frac{1}{1.3} - \frac{1}{3.5}\right) + \left(\frac{1}{3.5} - \frac{1}{5.7}\right) + ...\right]$

$= \frac{2}{3}$

11. From all the English alphabets, five letters are chosen and are arranged in alphabetical order. The total number of ways, in which the middle letter is 'M', is :

(1) 14950 (2) 6084

(3) 4356 (4) 5148

Ans. (4)

Sol. $\underbrace{AB\ \_\_\_\_}_{12}\ M\ \underbrace{N.........Z}_{13}$

$= \underbrace{{}^{12}C_2}_{\text{Selection of two letters before M}} \times \underbrace{{}^{13}C_2}_{\text{Selection of two letters after M}} = 5148$

12. Let $x = x(y)$ be the solution of the differential equation $y^2dx + \left(x - \frac{1}{y}\right)dy = 0$. If $x(1) = 1$, then $x\left(\frac{1}{2}\right)$ is :

(1) $\frac{1}{2} + e$ (2) $\frac{3}{2} + e$

(3) $3 - e$ (4) $3 + e$

Ans. (3)

Sol. $\frac{dx}{dy}+\left(\frac{1}{y^2}\right)x=\frac{1}{y^3}$

$\text{I.F.}=e^{\int\frac{1}{y^2}dy}=e^{-\frac{1}{y}}$

$\Rightarrow x.e^{-\frac{1}{y}}=\int\left(e^{-\frac{1}{t}}\right).\frac{1}{y^3}dy$

Put $-\frac{1}{y}=t$

$+\frac{1}{y^2}dy=dt$

$x.e^{-\frac{1}{y}}=-\int t.e^t dt$

$x.e^{-\frac{1}{y}}=-te^t+e^t+C$

$x.e^{-\frac{1}{y}}=\frac{+1}{y}e^{-\frac{1}{y}}+e^{-\frac{1}{y}}+C$

$x=1, y=1$

$\frac{1}{e}=\frac{1}{e}+\frac{1}{e}+C$

$\Rightarrow C=-\frac{1}{e}$

Put $y=\frac{1}{2}$

$\frac{x}{e^2}=\frac{2}{e^2}+\frac{1}{e^2}-\frac{1}{e}$

$x=3-e$

13. Let the parabola $y = x^2 + px - 3$, meet the coordinate axes at the points P, Q and R. If the circle C with centre at $(-1, -1)$ passes through the points P, Q and R, then the area of ΔPQR is :

(1) 4 (2) 6

(3) 7 (4) 5

Ans. (2)

Sol. $y=x^2+px-3$

Let $P(\alpha, 0)$, $Q(\beta, 0)$, $R(0, -3)$

Circle with centre $(-1, -1)$ is $(x+1)^2+(y+1)^2=r^2$

Passes through $(0, -3)$

$1^2+(-2)^2=r^2\,]$

$r^2=5$

$(x+1)^2+(y+1)^2=5$

Put $y=0$

$(x+1)^2=5-1$

$(x+1)^2=4$

$x+1=\pm 2$

$x=1$ or $x=-3$

$\therefore$ P(1, 0) and Q(–3,0)

Area of $\Delta PQR=\frac{1}{2}\begin{vmatrix}1 & 0 & 1\\ -3 & 0 & 1\\ 0 & -3 & 1\end{vmatrix}=6$

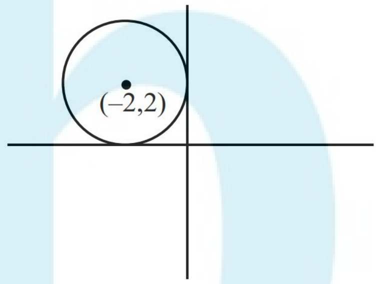
14. A circle C of radius 2 lies in the second quadrant and touches both the coordinate axes. Let r be the radius of a circle that has centre at the point (2, 5) and intersects the circle C at exactly two points. If the set of all possible values of r is the interval (α, β), then $3\beta - 2\alpha$ is equal to :

(1) 15 (2) 14

(3) 12 (4) 10

Ans. (1)

Sol.



$S_1 : (x+2)^2+(y-2)^2=2^2$

$S_2 : (x-2)^2+(y-5)^2=r^2$

Both circle intersect at two points

$\therefore |r_1-r_2|<c_1c_2<r_1+r_2$

$|r-2|<5<2+r$

$\Rightarrow 3<r<7$

$r\in(3, 7)$

$\alpha=3, \beta=7$

$3\beta-2\alpha=15$

15. Let for $f(x) = 7\tan^8x + 7\tan^6x - 3\tan^4x - 3\tan^2x$, $I_1=\int_0^{\pi/4}f(x)dx$ and $I_2=\int_0^{\pi/4}xf(x)dx$. Then $7I_1+12I_2$ is equal to :

(1) 2π (2) π

(3) 1 (4) 2

Ans. (3)

Sol. $f(x) = (7\tan x - 3\tan x)(\sec x)$

$$I_1 = \int_0^{\pi/4} (7\tan^6 x - 3\tan^2 x)(\sec^2 x)\,dx$$

Put tanx = t

$$I_1 = \int_0^1 (7t^6 - 3t^2)\,dt = \left[t^7 - t^3\right]_0^1 = 0$$

$$I_2 = \int_0^{\pi/4} \underset{I}{x}\underbrace{(7\tan^6 x - 3\tan^2 x)(\sec^2 x)}_{II}\,dx$$

$$= \left[x\left(\tan^7 x - \tan^3 x\right)\right]_0^{\pi/4} - \int_0^{\pi/4} (\tan^7 x - \tan^3 x)\,dx$$

$$= 0 - \int_0^{\pi/4} \tan^3 x\left(\tan^2 x - 1\right)\left(1 + \tan^2 x\right)dx$$

Put tanx = t

$$= -\int_0^1 \left(t^5 - t^3\right)dt = -\left[\frac{t^6}{6} - \frac{t^4}{4}\right] = \frac{1}{12}$$

$7I_1 + 12I_2 = 1$

16. Let $f(x)$ be a real differentiable function such that $f(0) = 1$ and $f(x + y) = f(x)f'(y) + f'(x)\,f(y)$ for all $x, y \in \mathbf{R}$. Then $\sum_{n=1}^{100} \log_e f(n)$ is equal to :

(1) 2384 (2) 2525

(3) 5220 (4) 2406

Ans. **(2)**

Sol. $f(x + y) = f(x)\,f'(y) + f'(x)\,f(x)$

Put = x = y = 0

$f(0) = f(0)f'(0) + f'(0)f(0)$

$f'(0) = \frac{1}{2}$

Put y = 0

$f(x) = f(x)\,f'(0) + f'(x)f(0)$

$$f(x) = \frac{1}{2}f(x) + f'(x)$$

$$f'(x) = \frac{f(x)}{2}$$

$$\frac{dy}{dx} = \frac{y}{2} \Rightarrow \int\frac{dy}{y} = \int\frac{dx}{2}$$

$$\Rightarrow \ell ny = \frac{x}{2} + c$$

$\because f(0) = 1 \Rightarrow C = 0$

$$\ell ny = \frac{\pi}{2} \Rightarrow f(x) = e^{x/2}$$

$$\ell n\, f(n) = \frac{n}{2}$$

$$\sum_{n=1}^{100} \ell f(n) = \frac{1}{2}\sum_{n=1}^{100} n = \frac{5050}{2}$$

$= 2525$

17. Let $A = \{1,2,3,.......,10\}$ and $B = \left\{\frac{m}{n} : m, n \in A,\ m < n \text{ and } \gcd(m, n) = 1\right\}$.

Then n(B) is equal to :

(1) 31 (2) 36

(3) 37 (4) 29

Ans. **(1)**

Sol. $A = \{1, 2, ...10\}$

$B\ \{\frac{m}{n} = m, n \in A, m < n, \gcd(m, n) = 1\}$

n(B)

n = 2 $\left\{\frac{1}{2}\right\}$

n = 3 $\left\{\frac{1}{3}, \frac{2}{3}\right\}$

n = 4 $\left\{\frac{1}{4}, \frac{3}{4}\right\}$

n = 5 $\left\{\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}\right\}$

n = 6 $\left\{\frac{1}{6}, \frac{5}{6}\right\}$

n = 7 $\left\{\frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}\right\}$

n = 8 $\left\{\frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8}\right\}$

n = 9 $\left\{\frac{1}{9}, \frac{2}{9}, \frac{4}{9}, \frac{5}{9}, \frac{7}{9}, \frac{8}{9}\right\}$

n = 10 $\left\{\frac{1}{10}, \frac{3}{10}, \frac{7}{10}, \frac{9}{10}\right\}$

n(B) = 31

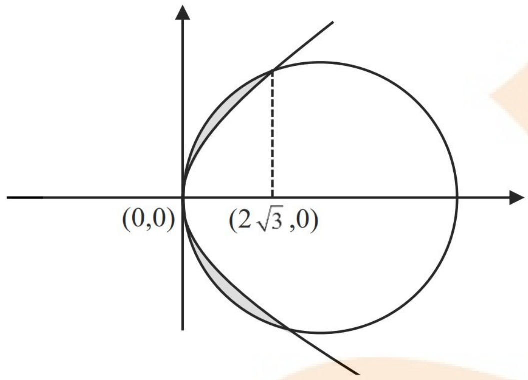
18. The area of the region, inside the circle $\left(x-2\sqrt{3}\right)^2+y^2=12$ and outside the parabola $y^2=2\sqrt{3}x$ is

(1) $6\pi-8$ (2) $3\pi-8$

(3) $6\pi-16$ (4) $3\pi+8$

Ans. **(3)**

Sol.



$y^2 = 2\sqrt{3}x$

$\left(x-2\sqrt{3}\right)^2+y^2=\left(2\sqrt{3}\right)^2$

$A=\frac{\pi r^2}{2}-2\int_0^{2\sqrt{3}}\sqrt{2\sqrt{3}x}\,dx$

$\frac{\pi(12)}{2}-2\sqrt{2\sqrt{3}}\frac{\left(x^{3/2}\right)_0^{2/3}}{3/2}$

$=6\pi-16$

19. Two balls are selected at random one by one without replacement from a bag containing 4 white and 6 black balls. If the probability that the first selected ball is black, given that the second selected ball is also black, is $\frac{m}{n}$, where gcd(m, n) = 1, then m + n is equal to :

(1) 14 (2) 4

(3) 11 (4) 13

Ans. **(1)**

Sol. $P=\frac{\frac{6}{10}\times\frac{5}{9}}{\frac{4}{10}\times\frac{6}{9}+\frac{6}{10}\times\frac{5}{9}}=\frac{5}{9}$

m = 5, n = 9

m + n = 14

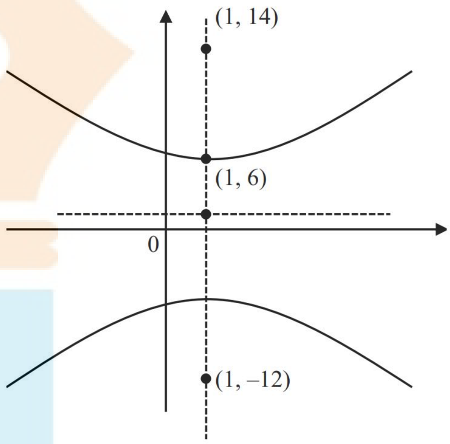
20. Let the foci of a hyperbola be (1, 14) and (1, –12). If it passes through the point (1, 6), then the length of its latus-rectum is :

(1) $\frac{25}{6}$ (2) $\frac{24}{5}$

(3) $\frac{288}{5}$ (4) $\frac{144}{5}$

Ans. **(3)**

Sol.



be = 13, b = 5

$a^2 = b^2 (e^2 – 1)$

$= b^2e^2 – b^2$

$= 169 – 25 = 144$

$\ell(LR)=\frac{2a^2}{b}=\frac{2\times144}{5}=\frac{288}{5}$

SECTION-B

21. Let the function,

$$f(x)=\begin{cases}-3ax^2-2, & x<1\\ a^2+bx, & x\ge1\end{cases}$$

Be differentiable for all x ∈ **R**, where a > 1, b ∈**R**. If the area of the region enclosed by y = f(x) and the line y = – 20 is $\alpha+\beta\sqrt{3}$, α, β,∈Z, then the value of α + β is _____.

Ans. **(34)**

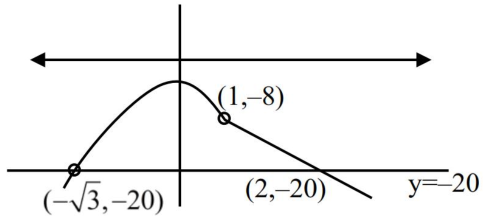
Sol. f(x) is continuous and differentiable

at x = 1; LHL = RHL, LHD = RHD

$-3a-2=a^2+b,\ -6a=b$

$a = 2, 1;\ b = -12$

$$f(x)=\begin{cases}-6x^2-2 & ; \ x<1\\ 4-12x & ; \ x\ge1\end{cases}$$



$$\text{Area} = \int_{-\sqrt{3}}^{1}\left(-6x^2-2+20\right)dx+\int_{1}^{2}(4-12x+20)\,dx$$

$$16+12\sqrt{3}+6=22+12\sqrt{3}$$

22. If $\sum_{r=0}^{5}\frac{{}^{11}C_{2r+1}}{2r+2}=\frac{m}{n}$, gcd(m, n) =1, then m – n is equal to ______.

Ans. **(2035)**

Sol. $\int_{0}^{1}(1+x)^{11}dx=\left[C_0x+\frac{C_1x^2}{2}+\frac{C_2x^3}{3}+\ldots\right]_0^1$

$$\frac{2^{12}-1}{12}=C_0+\frac{C_1}{2}+\frac{C_2}{3}+\frac{C_3}{4}+\ldots$$

$$\int_{-1}^{0}(1+x)^{11}dx=\left[C_0x+\frac{C_1x^2}{2}+\frac{C_2x^3}{3}+\ldots\right]_{-1}^{0}$$

$$\frac{1}{12}=C_0-\frac{C_1}{2}+\frac{C_2}{3}-\frac{C_3}{4}+\ldots$$

$$\frac{2^{12}-2}{12}=2\left(\frac{C_1}{2}+\frac{C_3}{4}+\frac{C_5}{6}+\ldots\right)$$

$$\frac{C_1}{2}+\frac{C_3}{4}-\frac{C_5}{6}+\ldots=\frac{2^{11}-1}{12}=\frac{2047}{12}$$

23. Let A be a square matrix of order 3 such that det(A) = –2 and det(3adj(–6adj(3A))) $=2^{m+n}.3^{mn}$, m > n. Then 4m + 2n is equal to ______.

Ans. **(34)**

Sol. $|A| = -2$

det(3adj(–6adj(3A)))

$= 3^3\det(\text{adj}(-\text{adj}(3A)))$

$= 3^3(-6)^6(\det(3A))^4$

$= 3^{21}\times 2^{10}$

m + n = 10

mn = 21

m = 7 ; n = 3

24. Let $L_1:\frac{x-1}{3}=\frac{y-1}{-1}=\frac{z+1}{0}$ and $L_2:\frac{x-2}{2}=\frac{y}{0}=\frac{z+4}{\alpha}$, $\alpha\in R$, be two lines, which intersect at the point B. If P is the foot of perpendicular from the point A(1, 1, –1) on L_2, then the value of $26\,\alpha(PB)^2$ is ____.

Ans. **(216)**

Sol. Point B

$(3\lambda+1, -\lambda+1, -1)\equiv(2\mu+2, 0, \alpha\mu-4)$

$3\lambda+1=2\mu+2$

$-\lambda+1=0$

$-1=\alpha\mu-4$

$\lambda=1, \mu=1, \alpha=3$

B(4, 0, –1)

Let Point ‘P’ is $(2\delta+2, 0, 3\delta-4)$

Dr’s of AP $< 2\delta+1, -1, 3\delta-3 >$

$AP\perp L_2\Rightarrow\delta=\frac{7}{13}$

$P\left(\frac{40}{13},0,\frac{-31}{13}\right)$

$2\sigma\delta(PB)^2=26\times3\times\left(\frac{144}{169}+\frac{324}{169}\right)$

$= 216$

25. Let $\vec{c}$ be the projection vector of $\vec{b}=\lambda\hat{i}+4\hat{k}$, $\lambda>0$, on the vector $\vec{a}=\hat{i}+2\hat{j}+2\hat{k}$. If $|\vec{a}+\vec{c}|$=7, then the area of the parallelogram formed by the vectors $\vec{b}$ and $\vec{c}$ is ______.

Ans. **(16)**

Sol. $\vec{c}=\left(\frac{\vec{b}\cdot\vec{a}}{|\vec{a}|}\right)\frac{\vec{a}}{|\vec{a}|}$

$=\left(\frac{\lambda+8}{9}\right)\left(\hat{i}+2\hat{j}+2\hat{k}\right)$

$|\vec{a}+\vec{c}|-=7\ \Rightarrow\lambda=4$

Area of parallelogram

$$=\left|\vec{b}\times\vec{c}\right|=\left|\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\ \frac{4}{3}&\frac{8}{3}&\frac{8}{3}\\ 4&0&4\end{vmatrix}\right|$$

$= 16$

PHYSICS

SECTION-A

26. Given below are two statements :

Statement I : In a vernier callipers, one vernier scale division is always smaller than one main scale division.

Statement II : The vernier constant is given by one main scale division multiplied by the number of vernier scale division.

In the light of the above statements, choose the **correct** answer from the options given below.

(1) Both **Statement I** and **Statement II** are false.

(2) **Statement I** is true but **Statement II** is false.

(3) Both **Statement I** and **Statement II** are true.

(4) **Statement I** is false but **Statement II** is true.

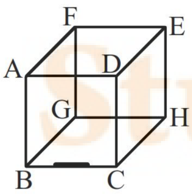
Ans. (2)

Sol. In general one vernier scale division is smaller than one main scale division but in some modified cases it may be not correct. Also least count is given by one main scale division / number of vernier scale division for normal vernier calliper.

Note: In JA-2016_P-2, Q-6 was present with modified V.C..

27. A line charge of length $\frac{'a'}{2}$ is kept at the center of an edge BC of a cube ABCDEFGH having edge length 'a' as shown in the figure. If the density of line is λC per unit length, then the total electric flux through all the faces of the cube will be ____.

(Take, $\in_0$ as the free space permittivity)



(1) $\frac{\lambda a}{8 \in_0}$ (2) $\frac{\lambda a}{16 \in_0}$

(3) $\frac{\lambda a}{2 \in_0}$ (4) $\frac{\lambda a}{4 \in_0}$

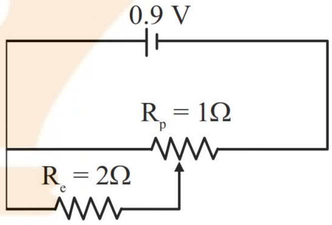
Ans. (1)

Sol. Total charge inside the cube

$$= \frac{\lambda \frac{a}{2}}{4} = \frac{\lambda a}{8}$$

$$\therefore \ \phi = \frac{q_{in}}{\varepsilon_0} = \frac{\lambda a}{8\varepsilon_0}$$

28.



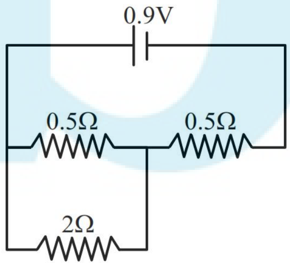
Sliding contact of a potentiometer is in the middle of the potentiometer wire having resistance R_p = 1Ω as shown in the figure. An external resistance of $R_e = 2\Omega$ is connected via the sliding contact.

(1) 0.3 A (2) 1.35 A

(3) 1.0 A (4) 0.9 A

Ans. (3)

Sol. The circuit can be considered as



$$\therefore \ R_{eq} = 0.5 + \frac{0.5 \times 2}{2 + 0.5} = \left(\frac{5}{10} + \frac{10}{25}\right)\Omega$$

$$= \frac{45}{50} = \frac{9}{10} = 0.9$$

$$\therefore \ i = \frac{0.9}{0.9} = 1A$$

29. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : If Young's double slit experiment is performed in an optically denser medium than air, then the consecutive fringes come closer.

Reason (R) : The speed of light reduces in an optically denser medium than air while its frequency does not change.

In the light of the above statements, choose the **most appropriate answer** from the options given below :

(1) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of **(A)**

(2) **(A)** is false but **(R)** is true.

(3) Both **(A)** and **(R)** are true but **(R)** is **not** the correct explanation of **(A)**

(4) **(A)** is true but **(R)** is false.

Ans. **(1)**

Sol. $\beta(\text{fringe width}) = \dfrac{\lambda D}{d}$

In denser medium, $\lambda \downarrow \Rightarrow \beta \downarrow$

$\Rightarrow$ fringe come closer

Also, $\mu = \dfrac{c}{V} \Rightarrow V = \dfrac{c}{\mu}$

Frequency remains same,

$\Rightarrow \mu = \dfrac{\lambda_{vac.} f}{\lambda_{med} f} \Rightarrow \lambda_{med} = \dfrac{\lambda_{vac.}}{\mu}$

30. Two spherical bodies of same materials having radii 0.2 m and 0.8 m are placed in same atmosphere. The temperature of the smaller body is 800 K and temperature of bigger body is 400 K. If the energy radiate from the smaller body is E, the energy radiated from the bigger body is (assume, effect of the surrounding to be negligible)

(1) 256 E (2) E

(3) 64 E (4) 16 E

Ans. **(2)**

Sol. $\dfrac{d\theta}{dt} = \sigma e A T^4 \Rightarrow P \propto AT^4$

$\dfrac{P_{smaller}}{P_{larger}} = \dfrac{(0.2)^2 \times 800^4}{(0.8)^2 \times 400^4}$

$\dfrac{1}{16} \times 16 = 1$

$\therefore P_{larger} = P_{smaller}$

31. An amount of ice of mass 10^{-3} kg and temperature $-10°C$ is transformed to vapour of temperature 110° by applying heat. The total amount of work required for this conversion is,

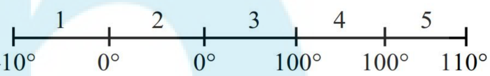
(Take, specific heat of ice = 2100 $Jkg^{-1}K^{-1}$, specific heat of water = 4180 $Jkg^{-1}K^{-1}$, specific heat of steam = 1920 $Jkg^{-1}K^{-1}$, Latent heat of ice = 3.35×10^5 Jkg^{-1} and Latent heat of steam = 2.25×10^6 Jkg^{-1})

(1) 3022 J (2) 3043 J

(3) 3003 J (4) 3024 J

Ans. **(2)**

Sol.



$\Delta Q_1 = m \times S_I \times \Delta T = 10^{-3} \times 2100 \times 10 = 21$ J

$\Delta Q_2 = m \times L_f = 10^{-3} \times 3.35 \times 10^5 = 335$ J

$\Delta Q_3 = m \times S_W \times \Delta T = 10^{-3} \times 4180 \times 100 = 418$ J

$\Delta Q_4 = m \times L_V = 10^{-3} \times 2.25 \times 10^6 = 2250$ J

$\Delta Q_5 = m \times S_V \times \Delta T = 10^{-3} \times 1920 \times 10 = 19.2$ J

$\Delta Q_{net} = 3043.2$ J

32. An electron in the ground state of the hydrogen atom has the orbital radius of 5.3×10^{-11} m while that for the electron in third excited state is 8.48×10^{-10} m. The ratio of the de Broglie wavelengths of electron in the ground state to that in excited state is

(1) 4 (2) 9

(3) 3 (4) 16

Ans. **(1)**

Sol. $\lambda = \dfrac{h}{mv}$

$$mvr = \frac{nh}{2\pi}$$

$$mv = \frac{nh}{2\pi r}$$

$$\lambda = \frac{2\pi rh}{nh}$$

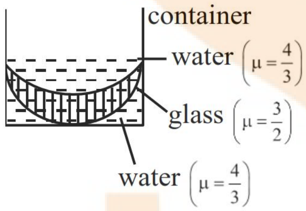
$$\lambda \propto \frac{r}{n}$$

$$\frac{\lambda_1}{\lambda_4} = \frac{r_1 n_4}{n_1 r_4} = \frac{5.3\times10^{-11}\times4}{1\times84.8\times10^{-11}}$$

$$\frac{\lambda_1}{\lambda_4} = \frac{1}{4}$$

Note : Most appropriate answer will be option (1).

33. In the diagram given below, there are three lenses formed. Considering negligible thickness of each of them as compared to $[R_1]$ and $[R_2]$, i.e., the radii of curvature for upper and lower surfaces of the glass lens, the power of the combination is



(1) $-\dfrac{1}{6}\left(\dfrac{1}{|R_1|}+\dfrac{1}{|R_2|}\right)$

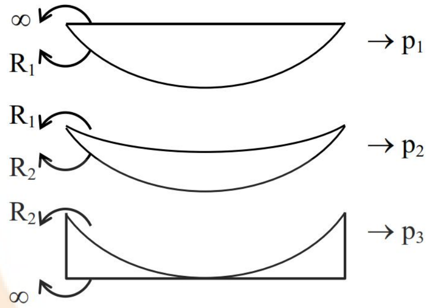
(2) $-\dfrac{1}{6}\left(\dfrac{1}{|R_1|}-\dfrac{1}{|R_2|}\right)$

(3) $\dfrac{1}{6}\left(\dfrac{1}{|R_1|}+\dfrac{1}{|R_2|}\right)$

(4) $\dfrac{1}{6}\left(\dfrac{1}{|R_1|}-\dfrac{1}{|R_2|}\right)$

Ans. (2)

Sol.



$$\Rightarrow p_{eq} = p_1 + p_2 + p_3$$

$$\Rightarrow p_1 = \left(\frac{4}{3}-1\right)\left(\frac{1}{\infty}-\frac{1}{-|R_1|}\right)$$

$$\Rightarrow p_1 = \left(\frac{1}{3|R_1|}\right)$$

$$\Rightarrow p_2 = \left(\frac{1}{2}\right)\left(\frac{1}{-|R_1|}-\frac{1}{-|R_2|}\right)$$

$$\Rightarrow p_2 = \frac{1}{2}\left(\frac{1}{|R_2|}-\frac{1}{|R_1|}\right)$$

$$\Rightarrow p_3 = \left(\frac{1}{3}\right)\left(\frac{1}{-|R_2|}-\frac{1}{\infty}\right) = -\frac{1}{3|R_2|}$$

$$\Rightarrow p_{eq} = \frac{1}{3}\left(\frac{1}{|R_1|}-\frac{1}{|R_2|}\right)-\frac{1}{2}\left(\frac{1}{|R_1|}-\frac{1}{|R_2|}\right)$$

$$= -\frac{1}{6}\left(\frac{1}{|R_1|}-\frac{1}{|R_2|}\right)$$

34. An electron is made to enters symmetrically between two parallel and equally but oppositely charged metal plates, each of 10 cm length. The electron emerges out of the field region with a horizontal component of velocity 10^6 m/s. If the magnitude of the electric between the plates is 9.1 V/cm, then the vertical component of velocity of electron is

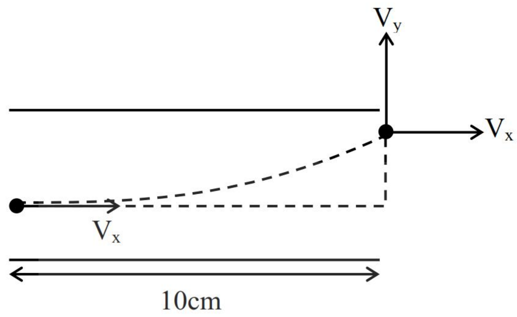
(mass of electron = 9.1×10^{-31} kg and charge of electron = 1.6×10^{-19} C)

(1) 1×10^6 m/s (2) 0

(3) 16×10^6 m/s (4) 16×10^4 m/s

Ans. (3)

Sol.



$$\Rightarrow t = \frac{\ell}{V_x} = \frac{10\times10^{-2}}{10^{6}} = 10^{-7}$$

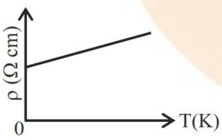
$$\Rightarrow V_y = u_y + a_y t$$

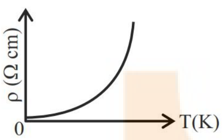
$$\Rightarrow V_y = 0 + \frac{eE}{m}\times10^{-7}$$

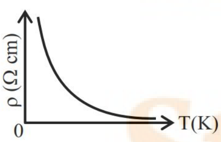
$$\Rightarrow V_y = \frac{1.6\times10^{-19}}{9.1\times10^{-31}}\times9.1\times10^{-2}\times10^{-7}$$

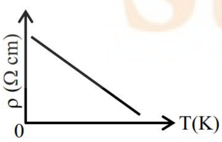
$$\Rightarrow V_y = 16\times10^{6}$$

35. Which of the following resistivity (ρ) v/s temperature (T) curves is most suitable to be used in wire bound standard resistors?

(1) 

(2) 

(3) 

(4) 

Ans. **(1)**

Sol. Resistivity is independent of temperature for wire bound resistors

36. A closed organ and an open organ tube filled by two different gases having same bulk modulus but different densities ρ_1 and ρ_2 respectively. The frequency of 9^{th} harmonic of closed tube is identical with 4^{th} harmonic of open tube. If the length of the closed tube is 10 cm and the density ratio of the gases is $\rho_1 : \rho_2 = 1 : 16$, then the length of the open tube is :

(1) $\frac{20}{7}$ cm (2) $\frac{15}{7}$ cm

(3) $\frac{20}{9}$ cm (4) $\frac{15}{9}$ cm

Ans. **(3)**

Sol. 9^{th} harmonic of closed pipe $= \frac{9V_1}{4\ell_1}$

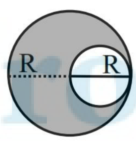
4^{th} harmonic of open pipe $= \frac{2V_2}{\ell_2}$

$$\therefore \frac{9V_1}{4\ell_1} = \frac{2V_2}{\ell_2}$$

$$\therefore \frac{9}{4\ell_1}\sqrt{\frac{B}{\rho_1}} = \frac{2}{\ell_2}\sqrt{\frac{B}{\rho_2}} \Rightarrow \frac{\ell_2}{\ell_1} = \frac{8}{9}\sqrt{\frac{\rho_1}{\rho_2}}$$

$$\ell_2 = \ell_1 \times \frac{8}{9}\times\frac{1}{4} = \frac{20}{9}\text{cm}$$

37. A uniform circular disc of radius 'R' and mass 'M' is rotating about an axis perpendicular to its plane and passing through its centre. A small circular part of radius R/2 is removed from the original disc as shown in the figure. Find the moment of inertia of the remaining part of the original disc about the axis as given above.



(1) $\frac{7}{32}MR^2$ (2) $\frac{9}{32}MR^2$

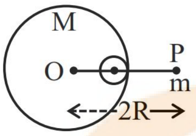
(3) $\frac{17}{32}MR^2$ (4) $\frac{13}{32}MR^2$

Ans. **(4)**

Sol. $I = \frac{MR^2}{2} - \left[\frac{\frac{M}{4}\left(\frac{R}{2}\right)^2}{2} + \frac{M}{4}\left(\frac{R}{2}\right)^2\right]$

$I = \frac{13}{32}MR^2$

38. A small point of mass m is placed at a distance 2R from the centre 'O' of a big uniform solid sphere of mass M and radius R. The gravitational force on 'm' due to M is F_1. A spherical part of radius R/3 is removed from the big sphere as shown in the figure and the gravitational force on m due to remaining part of M is found to be F_2. The value of ratio $F_1 : F_2$ is



(1) 16 : 9 (2) 11 : 10

(3) 12 : 11 (4) 12 : 9

Ans. **(3)**

Sol. $F_1 = \frac{GMm}{(2R)^2}$ ...(1)

$F_2 = \frac{GMm}{(2R)^2} - \left(\frac{G\left(\frac{M}{27}\right)m}{\left(\frac{4R}{3}\right)^2}\right)$

$F_2 = \frac{11}{48}\frac{GMm}{R^2}$ ...(2)

$F_1 : F_2 = 12 : 11$

39. The work functions of cesium (Cs) and lithium (Li) metals are 1.9 eV and 2.5 eV, respectively. If we incident a light of wavelength 550 nm on these two metal surface, then photo-electric effect is possible for the case of

(1) Li only (2) Cs only

(3) Neither Cs nor Li (4) Both Cs and Li

Ans. **(2)**

Sol. $E = \frac{1240}{\lambda} = \frac{1240}{550} \simeq 2.25$

40. If B is magnetic field and $_0$ is permeability of free space, then the dimensions of (B/μ_0) is

(1) $MT^{-2}A^{-1}$ (2) $L^{-1}A$

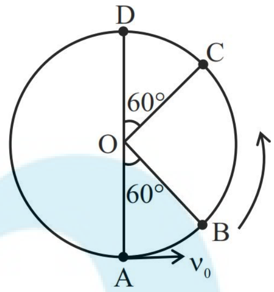
(3) $LT^{-2}A^{-1}$ (4) $ML^2T^{-2}A^{-1}$

Ans. **(2)**

Sol. $B = \mu_0 ni$

$\left[\frac{B}{\mu_0}\right] = [ni] = L^{-1}A^1$

41. A bob of mass m is suspended at a point O by a light string of length l and left to perform vertical motion (circular) as shown in figure. Initially, by applying horizontal velocity v_0 at the point 'A'. the string becomes slack when, the bob reaches at the point 'D'. The ratio of the kinetic energy of the bob at the points B and C is ______.



(1) 2 (2) 1

(3) 4 (4) 3

Ans. **(1)**

Sol. $\frac{1}{2}mv_A^2 = \frac{1}{2}mv_B^2 + mgh$

$\Rightarrow \frac{1}{2}m(5g\ell) = \frac{1}{2}mv_B^2 + mg\frac{\ell}{2}$

$\Rightarrow \frac{5mg\ell}{2} - \frac{mg\ell}{2} = KE_B$

$\Rightarrow KE_B = 2mg\ell$

$\frac{1}{2}mv_C^2 = \frac{1}{2}mv_D^2 + mg\frac{\ell}{2}$

$\Rightarrow KE_C = \frac{1}{2}mg\ell + mg\frac{\ell}{2} = mg\ell$

$\Rightarrow \frac{KE_B}{KE_C} = 2$

42. Given below are two statements :

Statement-I : The equivalent emf of two nonideal batteries connected in parallel is smaller than either of the two emfs.

Statement-II : The equivalent internal resistance of two nonideal batteries connected in parallel is smaller than the internal resistance of either of the two batteries.

In the light of the above statements, choose the **correct** answer from the options given below.

(1) **Statement-I** is true but **Statement-II** is false

(2) Both **Statement-I** and **Statement-II** are false

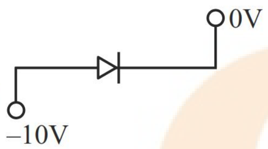
(3) Both **Statement-I** and **Statement-II** are true

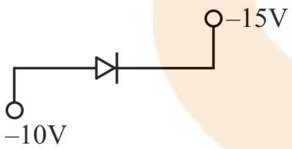
(4) **Statement-I** is false but **Statement-II** is true

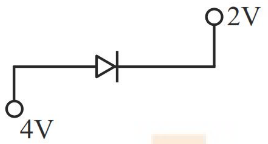
Ans. (4)

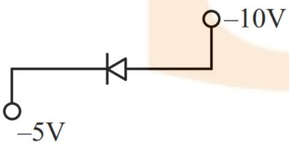
Sol. $= \dfrac{\dfrac{\varepsilon_1}{r_1}+\dfrac{\varepsilon_2}{r_2}}{\dfrac{1}{r_1}+\dfrac{1}{r_2}} = \varepsilon$

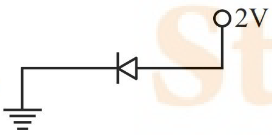
43. Which of the following circuits represents a forward biased diode ?

(A) 

(B) 

(C) 

(D) 

(E) 

Choose the **correct** answer from the options given below :

(1) (B), (D) and (E) only

(2) (A) and (D) only

(3) (B), (C) and (E) only

(4) (C) and (E) only

Ans. (3)

44. A parallel-plate capacitor of capacitance $40\mu F$ is connected to a 100 V power supply. Now the intermediate space between the plates is filled with a dielectric material of dielectric constant K = 2. Due to the introduction of dielectric material, the extra charge and the change in the electrostatic energy in the capacitor, respectively, are -

(1) 2 mC and 0.2 J (2) 8 mC and 2.0 J

(3) 4 mC and 0.2 J (4) 2 mC and 0.4 J

Ans. (3)

Sol. $\Delta q = (KC - C)V$

$= 40 \times 10^{-6} \times 100$

$= 4000 \times 10^{-3} = 4 \text{ mC}$

$\Delta U = \frac{1}{2}C'V^2 - \frac{1}{2}CV^2 = \frac{1}{2}(K-1)CV^2$

$= \frac{1}{2}CV^2(2-1)$

$= \frac{1}{2}CV^2 = \frac{1}{2} \times 40 \times 10^{-6} \times 10000$

$= 0.2 \text{ J}$

45. Given is a thin convex lens of glass (refractive index μ) and each side having radius of curvature R. One side is polished for complete reflection. At what distance from the lens, an object be placed on the optic axis so that the image gets formed on the object itself.

(1) R/μ (2) $R/(2\mu-3)$

(3) μR (4) $R/(2\mu-1)$

Ans. (4)

Sol. $P_{eq} = 2P_\ell + P_m$

$-\frac{1}{f_Q} = \frac{2}{f_\ell} - \frac{1}{f_m}$

$= \frac{4(\mu-1)}{R} - \frac{2}{-R} = \frac{1}{R}(4\mu - 4 + 2)$

$-\frac{1}{f_{eq}} = \frac{1}{R}(4\mu - 2)$

$\Rightarrow \frac{1}{f_{eq}} = \frac{-1}{R}(4\mu - 2)$

$f_{eq} = \frac{R}{2}$

$R = 2f_{eq} = -2\left(\frac{R}{4\mu-2}\right) = \frac{-R}{(2\mu-1)}$

SECTION-B

46. Two soap bubbles of radius 2 cm and 4 cm, respectively, are in contact with each other. The radius of curvature of the common surface, in cm, is ______ .

Ans. **(4)**

Sol. $r=\dfrac{r_1\cdot r_2}{r_1-r_2}=\dfrac{2\cdot 4}{4-2}=4\text{cm}$

47. The driver sitting inside a parked car is watching vehicles approaching from behind with the help of his side view mirror, which is a convex mirror with radius of curvature R = 2 m. Another car approaches him from behind with a uniform speed of 90 km/hr. When the car is at a distance of 24 m from him, the magnitude of the acceleration of the image of the side view mirror is 'a'. The value of 100a is ______ m/s^2.

Ans. **(8)**

Sol. $v=\dfrac{uf}{u-f}=\dfrac{-24\cdot 1}{-24-1}=\dfrac{24}{25}$

$m=\dfrac{-v}{u}=-\dfrac{24}{25(-24)}=\dfrac{1}{25}$

$\dfrac{1}{v}-\dfrac{1}{u}=\dfrac{1}{f}$

$v_I=-m^2\cdot v_0=\dfrac{-1}{(25)^2}\cdot 25=\dfrac{-1}{25}$

Diff.

$\dfrac{-1}{v^2}\left(\dfrac{dv}{dt}\right)+\dfrac{1}{u^2}\left(\dfrac{du}{dt}\right)=0\quad\left[\dfrac{dv}{dt}=v_I;\dfrac{du}{dt}=v_0\right]$

$\dfrac{+2}{v^3}\cdot(v_I)^2-\dfrac{1}{v^2}\cdot a_I-\dfrac{2}{u^3}\cdot(v_0)^2+\dfrac{1}{u^2}\cdot a_0=0$

$\dfrac{a_I}{v^2}=\dfrac{2}{v^3}\cdot v_I^2-\dfrac{2}{u^3}\cdot v_0^2$

$a_I=\dfrac{2}{v}\cdot v_I^2-\dfrac{2v^2}{u^3}\cdot v_0^2$

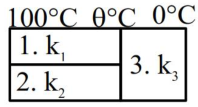
$=\dfrac{2\cdot 25}{24}\cdot\dfrac{1}{25}\cdot\dfrac{1}{25}-\dfrac{2}{(24)^3}\cdot\dfrac{24}{25}\cdot\dfrac{24}{25}\cdot 25\cdot 25$

$a_I=\dfrac{2}{24.25}-\dfrac{2}{24}$

$a_I=\dfrac{2}{24}\cdot\dfrac{-24}{25}=\dfrac{-2}{25}$

$100a_I=\dfrac{2}{25}\times 100=8$

48. Three conductions of same length having thermal conductivity k_1, k_2 and k_3 are connected as shown in figure.

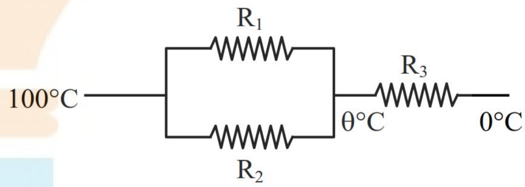


Area of cross sections of 1st and 2nd conductor are same and for 3rd conductor it is double of the 1st conductor. The temperatures are given in the figure. In steady state condition, the value of θ is ______ °C.

(Given : $k_1 = 60\ Js^{-1}m^{-1}K^{-1}$, $k_2 = 120\ Js^{-1}m^{-1}K^{-1}$, $k_3 = 135\ Js^{-1}m^{-1}K^{-1}$)

Ans. **(40)**

Sol.



$R_1=\dfrac{2L}{K_1A}$

$R_2=\dfrac{2L}{K_2A}$

$R_3=\dfrac{L}{K_3A}$

$\dfrac{\theta-100}{\dfrac{R_1R_2}{R_1+R_2}}+\dfrac{\theta-0}{R_3}=0$

$\theta=40$

49. The position vectors of two 1 kg particles, (A) and (B), are given by

$\vec{r}_A=\left(\alpha_1t^2\hat{i}+\alpha_2t\hat{j}+\alpha_3t\hat{k}\right)$ m

and $\vec{r}_B=\left(\beta_1t\hat{i}+\beta_2t^2\hat{j}+\beta_3t\hat{k}\right)$m, respectively ;

($\alpha_1 = 1\ m/s^2$, $\alpha_2 = 3n$ m/s, $\alpha_3 = 2$ m/s, $\beta_1 = 2$ m/s, $\beta_2 = -1\ m/s^2$, $\beta_3 = 4p$ m/s), where t is time, n and p are constants, At t = 1s, $|\vec{V}_A|=|\vec{V}_B|$ and velocities $\vec{V}_A$ and $\vec{V}_B$ of the particles are orthogonal to each other. At t = 1 s, the magnitude of angular momentum of particle (A) with respect to the position of particle (B) is $\sqrt{L}$ kgm^2s^{-1}. The value of L is _____.

Ans. **(90)**

Sol. $V_A = (2t\hat{i} + 3n\hat{j} + 2\hat{k})$

$\vec{V}_B = (2\hat{i} - 2t\hat{j} + 4p\hat{k})$

$\vec{V}_A \cdot \vec{V}_B = 0$

$4 - 6n + 8p = 0$

$2 - 3n + 4p = 0$

$3n = 2 + 4p$

$|\vec{V}_A| = |\vec{V}_B|$

$4 + 9n^2 + 4 = 4 + 4 + 16p^2$

$p = \frac{-1}{4} \quad \Rightarrow n = \frac{1}{3}$

$\vec{L} = m_A\left(\vec{r}_{A/B} \times \vec{V}_A\right)$

$\vec{r}_{A/B} = (\alpha_1 - \beta_1)\hat{i} + (\alpha_2 - \beta_2)\hat{j} + (\alpha_3 - \beta_3)$

$= (1-2)\hat{i} + (1+1)\hat{j} + 3\hat{k}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 3 \\ 2 & 1 & 2 \end{vmatrix} = \hat{i} + 8\hat{j} - 5\hat{k}$$

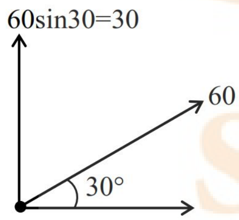
$= \sqrt{1+64+25} = \sqrt{90}$

50. A particle is projected at an angle of 30° from horizontal at a speed of 60 m/s. The height traversed by the particle in the first second is h_0 and height traversed in the last second, before it reaches the maximum height, is h_1. The ratio $h_0 : h_1$ is ______.

[Take, $g = 10\ m/s^2$]

Ans. **(5)**

Sol.



$S_1 = 30 \times 1 - \frac{1}{2} \times 10 \times 1 = 25$

$S_3 = 30 + \left(\frac{-10}{2}\right) \times (2 \times 3 - 1) = 5$

$\frac{S_1}{S_3} = \frac{25}{5} = 5$

CHEMISTRY

SECTION-A

51. A solution of aluminium chloride is electrolysed for 30 minutes using a current of 2A. The amount of the aluminium deposited at the cathode is ____ .
[Given : molar mass of aluminium and chlorine are 27 g mol^{-1} and 35.5 g mol^{-1} respectively, Faraday constant = 96500 C mol^{-1}].

(1) 1.660 g (2) 1.007 g
(3) 0.336 g (4) 0.441 g

Ans. **(3)**

Sol. gm equivalent of Al deposited = $\frac{It}{96500}$

$$\frac{w}{27}\times 3 = \frac{2\times 30\times 60}{96500}$$

w = 0.336 g,

52. Which of the following statement is not true for radioactive decay ?

(1) Amount of radioactive substance remained after three half lives is $\frac{1}{8}$ th of original amount.

(2) Decay constant does not depend upon temperature.

(3) Decay constant increases with increase in temperature.

(4) Half life is ln 2 times of $\frac{1}{\text{rate constant}}$.

Ans. **(3)**

Sol. Decay constant is independent of temperature.

53. How many different stereoisomers are possible for the given molecule ?

$$\begin{array}{l} CH_3-CH-CH=CH-CH_3 \\ \qquad\;\; | \\ \qquad\; OH \end{array}$$

(1) 3 (2) 1
(3) 2 (4) 4

Ans. **(4)**

Sol. $\begin{array}{l} CH_3-CH-CH=CH-CH_3 \\ \qquad\;\; | \\ \qquad\; OH \end{array}$

It has 4 stereoisomers $\begin{bmatrix} \text{R cis} & \text{R trans} \\ \text{S cis} & \text{S trans} \end{bmatrix}$

54. Which of the following electronegativity order is **incorrect**?

(1) Al < Mg < B < N (2) Al < Si < C < N
(3) Mg < Be < B < N (4) S < Cl < O < F

Ans. **(1)**

Sol.

	Li	Be	B	C	N	O	F
(E.N.)=	1	1.5	2	2.5	3	3.5	4.0

On pauling scale

	Na	Mg	Al	Si	P	S	Cl
(E.N.)=	0.9	1.2	1.5	1.8	2.1	2.5	3.0

Correct order Mg < Al < B < N

55. Lanthanoid ions with $4f^7$ configuration are :

(A) Eu^{2+} (B) Gd^{3+} (C) Eu^{3+} (D) Tb^{3+}
(E) Sm^{2+}

Choose the correct answer from the options given below :

(1) (A) and (B) only (2) (A) and (D) only
(3) (B) and (E) only (4) (B) and (C) only

Ans. **(1)**

Sol. ${}_{63}Eu^{2+}$ – [Xe] $4f^7 6s^0$

${}_{64}Gd^{3+}$ – [Xe] $4f^7\ 5d^0 6s^0$

${}_{63}Eu^{3+}$ – [Xe] $4f^6\ 6s^0$

${}_{65}Tb^{3+}$ – [Xe] $4f^8\ 6s^0$

${}_{62}Sm^{2+}$ – [Xe] $4f^6\ 6s^0$

Eu^{2+} & Gd^{3+}

56. Match **List-I** with **List-II**

List-I		List-II	
(A)	$Al^{3+} < Mg^{2+} < Na^{+} < F^{-}$	(I)	Ionisation Enthalpy
(B)	$B < C < O < N$	(II)	Metallic character
(C)	$B < Al < Mg < K$	(III)	Electronegativity
(D)	$Si < P < S < Cl$	(IV)	Ionic radii

Choose the **correct** answer from the options given below :

(1) A-IV, B-I, C-III, D-II (2) A-II, B-III, C-IV, D-I

(3) A-IV, B-I, C-II, D-III (4) A-III, B-IV, C-II, D-I

Ans. (3)

Sol. Ionic radii – $Al^{3+} < Mg^{2+} < Na^{+} < F^{-}$

Ionisation energy – $B < C < O < N$

Metallic character – $B < Al < Mg < K$

Electron negativity – $Si < P < S < Cl$

57. Which of the following acids is a vitamin ?

(1) Adipic acid (2) Aspartic acid

(3) Ascorbic acid (4) Saccharic acid

Ans. (3)

Sol. Vitamin-C is Ascorbic acid.

58. A liquid when kept inside a thermally insulated closed vessel at 25°C was mechanically stirred from outside. What will be the correct option for the following thermodynamic parameters ?

(1) $\Delta U > 0, q = 0, w > 0$ (2) $\Delta U = 0, q = 0, w = 0$

(3) $\Delta U < 0, q = 0, w > 0$ (4) $\Delta U = 0, q < 0, w > 0$

Ans. (1)

Sol. Thermally insulated $\Rightarrow q = 0$

from I^{st} law

$\Delta U = q + w$

$\Delta U = w$

$w > 0, \Delta U > 0$

59. Radius of the first excited state of Helium ion is given as :

$a_0 \rightarrow$ radius of first stationary state of hydrogen atom.

(1) $r = \frac{a_0}{2}$ (2) $r = \frac{a_0}{4}$ (3) $r = 4a_0$ (4) $r = 2a_0$

Ans. (4)

Sol. $r = a_0 \frac{n^2}{Z} = a_0 \cdot \frac{(2)^2}{2} = 2a_0.$

60. Given below are two statements :

Statement I : $CH_3 - O - CH_2 - Cl$ will undergo S_N1 reaction though it is a primary halide.

Statement II : $CH_3-\underset{CH_3}{\overset{CH_3}{\underset{|}{\overset{|}{C}}}}-CH_2-Cl$ will not undergo S_N2 reaction very easily though it is a primary halide.

In the light of the above statements, choose the **most appropriate answer** from the options given below :

(1) **Statement I** is incorrect but **Statement II** is correct.

(2) Both **Statement I** and **Statement II** are incorrect

(3) **Statement I** is correct but **Statement II** is incorrect

(4) Both **Statement I** and **Statement II** are correct.

Ans. (4)

Sol. $CH_3–O–CH_2–Cl$ will undergo S_N1 mechanism because $CH_3-O-\overset{+}{C}H_2$ is highly stable.

$H_3C-\underset{CH_3}{\overset{CH_3}{\underset{|}{\overset{|}{C}}}}-CH_2-Cl$ (Neopentyl chloride) will undergo S_N2 mechanism at a slow rate because it's sterically crowded

61. Given below are two statements :

Statement I : One mole of propyne reacts with excess of sodium to liberate half a mole of H_2 gas.

Statement II : Four g of propyne reacts with $NaNH_2$ to liberate NH_3 gas which occupies 224 mL at STP.

In the light of the above statements, choose the **most appropriate answer** from the options given below:

(1) **Statement I** is correct but **Statement II** is incorrect.
(2) Both **Statement I** and **Statement II** are incorrect
(3) **Statement I** is incorrect but **Statement II** is correct
(4) Both **Statement I** and **Statement II** are correct.

Ans. (1)

Sol.

$$\underset{1\text{mole}}{CH_3-C\equiv CH}+\underset{(\text{excess})}{Na}\rightarrow CH_3-C\equiv \overset{-}{C}\overset{+}{Na}+\underset{\frac{1}{2}\text{mole}\,H_2}{\frac{1}{2}H_2\uparrow}$$

$$CH_3-C\equiv CH+NaNH_2\rightarrow CH_3C\equiv \overset{-}{C}\overset{+}{Na}\;+\;NH_3$$

4 gm

$$\frac{4}{40}=0.1\text{mole} \qquad\qquad \frac{0.1\text{mole}}{2240\,\text{mole}}$$

Statement I is correct but **Statement II** is incorrect

62. A vessel at 1000 K contains CO_2 with a pressure of 0.5 atm. Some of CO_2 is converted into CO on addition of graphite. If total pressure at equilibrium is 0.8 atm, then K_P is :

(1) 0.18 atm (2) 1.8 atm (3) 0.3 atm (4) 3 atm.

Ans. (2)

Sol. $CO_2(g)+C(s) \rightleftharpoons 2CO(g)$

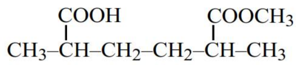
0.5   –
0.5–x   2x

$P_{total}=0.5+x=0.8$

$x=0.3$

$$K_P=\frac{(0.6)^2}{0.2}=1.8$$

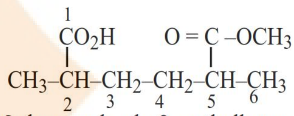
63. The IUPAC name of the following compound is :



(1) 2-Carboxy-5-methoxycarbonylhexane.
(2) Methyl-6-carboxy-2,5-dimethylhexanoate.
(3) Methyl-5-carboxy-2-methylhexanoate.
(4) 6-Methoxycarbonyl-2,5-dimethylhexanoic acid.

Ans. (4)

Sol.



5-Methoxycarbonly-2-methylhexanoic acid

64. Which of the following electrolyte can be sued to obtain $H_2S_2O_8$ by the process of electrolysis?

(1) Dilute solution of sodium sulphate
(2) Dilute solution of sulphuric acid
(3) Concentrated solution of sulphuric acid
(4) Acidified dilute solution of sodium sulphate.

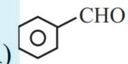
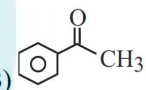
Ans. (3)

Sol. Theory based.

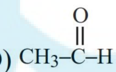
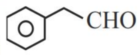
At anode :

$2HSO_4^- \rightarrow H_2S_2O_8+2e^-$

65. The compounds which give positive Fehling's test are :

(A)  (B) 

(C) $HOCH_2–CO–(CHOH)_3–CH_2–OH$

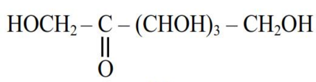
(D)  (E) 

Choose the **CORRECT** answer from the options given below :

(1) (A),(C) and (D) Only (2) (A),(D) and (E) Only
(3) (C), (D) and (E) Only (4) (A), (B) and (C) Only

Ans. (3)

Sol. $\underset{(C)}{CH_3CH=O}$, $\underset{(D)}{PhCH_2CH=O}$,



(E)

All gives positive Fehling test

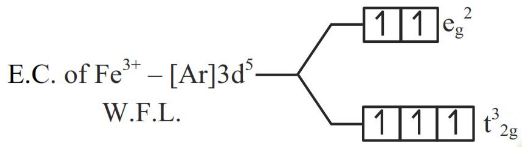
66. In which of the following complexes the CFSE, Δ_0 will be equal to zero?

(1) $[Fe(NH_3)_6]Br_2$ (2) $[Fe(en)_3]Cl_3$

(3) $K_4[Fe(CN)_6]$ (4) $K_3[Fe(SCN)_6]$

Ans. **(4)**

Sol. For complex $K_3[Fe(SCN)_6]$



Calculation of CFSE

$= (-0.4 \times 3 + 0.6 \times 2)\ \Delta_0$

$= 0\ \Delta_0$

67. Arrange the following solutions in order of their increasing boiling points.

(i) 10^{-4} M NaCl (ii) 10^{-4} M Urea

(iii) 10^{-3} M NaCl (iv) 10^{-2} M NaCl

(1) (ii) < (i) < (iii) < (iv) (2) (ii) < (i) $\cong$ (iii) < (iv)

(3) (i) < (ii) < (iii) < (iv) (4) (iv) < (iii) < (i) < (ii)

Ans. **(1)**

Sol. $\Delta T_b = i\,K_b\,.\,m\,. \propto i.C.$

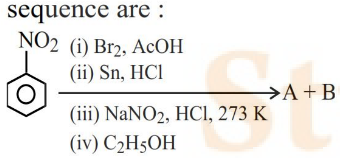
where C = concentration

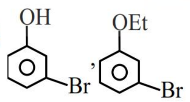
Options	i.C.
(i)	2×10^{-4}
(ii)	1×10^{-4}
(iii)	2×10^{-3}
(iv)	2×10^{-2}

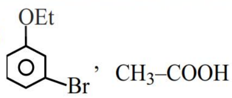
B.P. order :

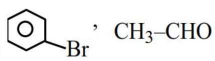
(ii) < (i) < (iii) < (iv)

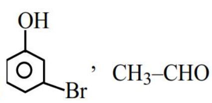
68. The products formed in the following reaction sequence are :



(1) 

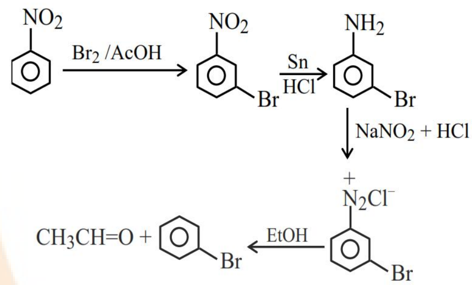
(2) 

(3) 

(4) 

Ans. **(3)**

Sol.



69. From the magnetic behaviour of $[NiCl_4]^{2-}$ (paramagnetic) and $[Ni(CO)_4]$ (diamagnetic), choose the correct geometry and oxidation state.

(1) $[NiCl_4]^{2-}$: Ni^{II}, square planar

$[Ni(CO)_4]$: Ni(0), square planar

(2) $[NiCl_4]^{2-}$: Ni^{II}, tetrahedral

$[Ni(CO)_4]$: Ni(0), tetrahedral

(3) $[NiCl_4]^{2-}$: Ni^{II}, tetrahedral

$[Ni(CO)_4]$: Ni^{II}, square planar

(4) $[NiCl_4]^{2-}$: Ni(0), tetrahedral

$[Ni(CO)_4]$: Ni(0), square planar

Ans. **(2)**

Sol. $[NiCl_4]^{2-}$

Ni^{+2} – [Ar] $3d^8\ 4s^0 \rightarrow sp^3$, Tetrahedral

Number of unpaired electron = 2 paramagentic

$[Ni(CO)_4]$,

Ni(0) $\rightarrow$ [Ar] $3d^{10}\ 4s^0$ (After rearrangement)

No unpaired electron

sp^3, Tetrahedral, Diamagnetic

70. The **incorrect** statements regarding geometrical isomerism are :

(A) Propene shows geometrical isomerism.

(B) Trans isomer has identical atoms/groups on the opposite sides of the double bond.

(C) Cis-but-2-ene has higher dipole moment than trans-but-2-ene.

(D) 2-methylbut-2-ene shows two geometrical isomers.

(E) Trans-isomer has lower melting point that cis isomer.

Choose the **CORRECT** answer from the options given below :

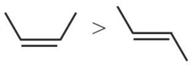
(1) (A), (D) and (E) only (2) (C), (D) and (E) only

(3) (B) and (C) only (4) (A) and (E) only

Ans. **(1)**

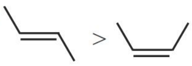
Sol. (A) $CH_3–CH=CH_2$. GI is not possible

(B) Trans isomer has identical atoms/groups on the opposite side of double bond.

(C)  (dipole moment only)

(D) $H_3C–\underset{\displaystyle CH_3}{\underset{|}{C}}=CH–CH_3$ (does not show GI)

2-methylbut-2-ene

(E)  (Melting point)

SECTION-B

71. Some CO_2 gas was kept in a sealed container at a pressure of 1 atm and at 273 K. This entire amount of CO_2 gas was later passed through an aqueous solution of $Ca(OH)_2$. The excess unreacted $Ca(OH)_2$ was later neutralized with 0.1 M of 40 mL HCl. If the volume of the sealed container of CO_2 was x, then x is ______ cm^3 (nearest integer).

[Given : The entire amount of $CO_2(g)$ reacted with exactly half the initial amount of $Ca(OH)_2$ present in the aqueous solution.]

Ans. **(45)**

Sol. Let moles of CO_2 = n

moles of $Ca(OH)_2$ total initially = 2n

excess $Ca(OH)_2$ = n

gm equivalent of $Ca(OH)_2$ = gm equivalent of HCl

$$n \times 2 = 0.1 \times \frac{40}{1000} \times 1$$

$$n = 2 \times 10^{-3}$$

Volume of $CO_2 = 2 \times 10^{-3} \times 22400 = 44.8\ cm^3$

72. In Carius method for estimation of halogens, 180 mg of an organic compound produced 143.5 mg of AgCl. The percentage composition of chlorine in the compound is _______ %.

[Given : molar mass in g mol^{-1} of Ag : 108, Cl = 35.5]

Ans. **(20)**

Sol. $n_{Cl} = n_{AgCl} = \dfrac{143.5\times10^{-3}}{143.5} = 10^{-3}$

$$\% \ Cl = \frac{10^{-3}\times35.5}{180\times10^{-3}} \times 100 = 19.72$$

73. The number of molecules/ions that show linear geometry among the following is _______ .

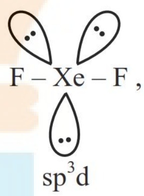
SO_2, $BeCl_2$, CO_2, N_3^-, NO_2, F_2O, XeF_2, NO_2^+, I_3^-, O_3

Ans. **(6)**

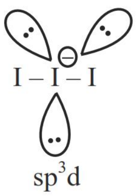
Sol. Linear species are

$Cl – Be – Cl$, $O = C = O$, $N^- = N^+ = N^-$

(sp) (sp) (sp)



$\overset{+}{O = N = O}$,
sp



74. $A \rightarrow B$

The molecule A changes into its isomeric form B by following a first order kinetics at a temperature of 1000 K. If the energy barrier with respect to reactant energy for such isomeric transformation is 191.48 kJ mol^{-1} and the frequency factor is 10^{20}, the time required for 50%, molecules of A to become B is ______ picoseconds (nearest integer).

[R = 8.314 J K^{-1} mol^{-1}]

Ans. **(69)**

Sol. $t_{1/2} = \dfrac{0.693}{K}$

$K = Ae^{-Ea/RT}$

$$= 10^{20} \times e^{-\frac{191.48\times10^3}{8.314\times1000}}$$

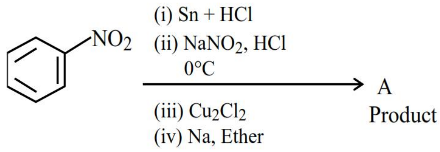
$$= 10^{20} \times e^{-23.031} = 10^{20} \times – e^{\ln 10 \times 10}$$

$$= \frac{10^{20}}{10^{10}} = 10^{10}\ sec.$$

$$t_{1/2} = \frac{0.693}{10^{10}} = 6.93 \times 10^{-11}$$

$$= 69.3 \times 10^{-12}\ sec.$$

75. Consider the following sequence of reactions :



Molar mass of the product formed (A) is ________ g mol^{-1} .

Ans. (154)

Sol.